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Foreword

Welcome to ESTEEM Volume 2. In this issue, we address a gamut of topics from the engineering disciplines to language education. We hope that ESTEEM, by publishing articles from a diverse range of disciplines, will encourage debate and exchange among researchers from assorted academic backgrounds.

I would like to thank our advisor, Prof. Madya Mohd Zaki Abdullah for his distinctive imprint on this edition. His leadership of the journal in its 2nd year of growing impact and reputation has been outstanding. His vision, commitment to excellence, and attention to detail are widely recognized by the Penang academic community as determining factors in the journal's success so far. We will do our best to continue and expand on this tradition of excellence.

Since its launch in 2003, ESTEEM is indeed fortunate to have a dynamic Editorial Team. These people have provided the journal with an outstanding service of reviewing submissions for publications. The journal follows the established policy of a blind review process consisting of at least two peer reviewers per submission. We depend upon their knowledge and judgement in advancing the scope and utility of this journal. Without their support and enthusiasm none of this would have been possible. Also, my thanks to all the contributors, both the successful and not so successful.

Our vision of the ***ESTEEM*** journal is that it should be the journal that belongs to you, the academic and research community. This includes all engineers and academicians working to unravel the mysteries of research, teaching and learning, in all its facets. We wish the journal to be responsive to your needs and your interests. Please feel free to contact any of us in the editorial board to give us your ideas and suggestions for the development of the journal. We look forward to working with you all in expanding this emerging venue for communicating high quality research on the many aspects of academia.

Finally, I would like to take this opportunity to invite all authors and readers to contact me at **esteem@ppinang.uitm.edu.my** to share their comments and advice on how to further enhance the journal's value to the wider research community in knowledge and how to move ESTEEM to the next level of excellence.

The Chief Editor
May, 2005

A New Finite Difference Scheme Based on Centroidal Mean Averaging for the Goursat Problem

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ABSTRACT

The Goursat partial differential equation arises in the simulation and modeling of several physical problems. Several finite difference schemes have been formulated for the Goursat equation. In this paper we develop a new finite difference scheme based on centroidal mean averaging of functional values and apply the scheme to solve two Goursat problems.

Keywords: *Goursat problem, Finite difference schemes, Centroidal mean*

Introduction

The Goursat problem is an initial value problem involving a hyperbolic partial differential equation which arises in several areas of physics and engineering. The standard numerical procedure is the use of a finite difference scheme with arithmetic mean averaging of functional values (Wazwaz, 1993). Wazwaz (1993), introduced a scheme based on harmonic mean averaging of functional values. Evans & Sanugi (1988), introduced a scheme based on geometric mean averaging of functional values. Comparative studies of these schemes have been carried out by Wazwaz (1993) as well as Ismail & Nasir (2004). In general the arithmetic mean scheme produces more accurate results and for linear problems it has an advantage in that the linearity of the scheme is preserved (i.e. no iterations are required).

In this paper we develop a scheme based on centroidal mean averaging of functional values (Centroidal Scheme), study its implementation and compare the results with that of a scheme based on arithmetic mean averaging of functional values (Arithmetic Scheme) on two Goursat problems.

The Centroidal Mean and Finite Difference Scheme

Let a and c be two natural numbers. Then the centroidal mean of a and c is (Faradj, 2004):

$$b = \frac{2(a^2 + ac + c^2)}{3(a+c)} \tag{1}$$

The Goursat problem is of the form:

$$\begin{aligned} &u_{xy} = f(x, y, u, u_x, u_y) \\ &u(x,0) = \phi(x),\ u(0,y) = \psi(y),\ \phi(0) = \psi(0) \\ &0 \le x \le a,\ 0 \le y \le b \end{aligned} \tag{2}$$

We consider the Goursat partial differential equations $u_{xy} = f$. The L.H.S is discretized as follows:

$$\begin{aligned} \frac{\partial^2 u}{\partial x \partial y} &= \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) \\ &= \frac{\partial}{\partial y}\left(\frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}\right) \\ &= \frac{1}{\Delta x}\frac{\partial}{\partial y}\big(u(x+\Delta x, y) - u(x,y)\big) \\ &= \frac{1}{\Delta x}\left[\frac{(u(x+\Delta x, y+\Delta y) - u(x+\Delta x, y))}{\Delta y} - \frac{(u(x, y+\Delta y) - u(x,y))}{\Delta y}\right] \\ &= \frac{1}{h^2}\big[u(x+\Delta x, y+\Delta y) - u(x+\Delta x, y)) - u(x, y+\Delta y) + u(x,y)\big] \end{aligned}$$

$$\text{where } h = \Delta x = \Delta y \tag{3}$$

Hence equation (3) becomes:

$$u_{xy} = \frac{u_{i+1,j+1} + u_{i,j} - u_{i+1,j} - u_{i,j+1}}{h^2} \tag{4}$$

The R.H.S is approximated at (i + 1/2, j + 1/2) as follows:

$f_{i+\frac{1}{2},j+\frac{1}{2}}$ = Centroidal mean (Centroidal mean $f_{i+1,j}$ and $f_{i,j+1}$; Centroidal mean $f_{i+1,j+1}$ and $f_{i,j}$) (5)

From (5) we obtain:

$$f_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{4}{9}\left[\frac{\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)^2+\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)+\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)^2}{\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)+\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)}\right] \tag{6}$$

Thus the Centroidal Scheme for the Goursat problem is given by:

$$\frac{u_{i+1,j+1}+u_{i,j}-u_{i+1,j}-u_{i,j+1}}{h^2} = \frac{4}{9}\left[\frac{\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)^2+\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)+\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)^2}{\left(\frac{f_{i+1,j+1}^2+f_{i+1,j+1}f_{i,j}+f_{i,j}^2}{f_{i+1,j+1}+f_{i,j}}\right)+\left(\frac{f_{i+1,j}^2+f_{i+1,j}f_{i,j+1}+f_{i,j+1}^2}{f_{i+1,j}+f_{i,j+1}}\right)}\right] \tag{7}$$

The Arithmetic Scheme for the Goursat problem is given by Wazwaz (1993):

$$\frac{u_{i+1,j+1}+u_{i,j}-u_{i+1,j}-u_{i,j+1}}{h^2} = \frac{1}{4}\left(f_{i+1,j+1}+f_{i,j}+f_{i+1,j}+f_{i,j+1}\right) \tag{8}$$

Here the L.H.S of is "discretized" as before and the "discretization" of the R.H.S is obtained as follows:

$f_{i+\frac{1}{2},j+\frac{1}{2}}$ = Arithmetic mean (Arithmetic mean $f_{i+1,j}$ and $f_{i,j+1}$;

Arithmetic mean $f_{i+1,j+1}$ and $f_{i,j}$)

Numerical Experiments

We consider the non linear Goursat problem:

$$u_{xy} = e^{2u}$$

$$u(x,0) = \frac{x}{2} - \ln(1+e^{x})$$

$$u(0,y) = \frac{y}{2} - \ln(1+e^{y})$$

$$0 \le x \le 4, 0 \le y \le 4 \qquad (9)$$

The analytical solution for (9) can be found in Wazwaz (1993).

A computer program to implement the Centroidal Scheme was developed. For the non linear Goursat problem (9) with h = 0.05, we obtained

Relative Errors for Centroidal Scheme

X

Y	1.0	2.0	3.0	4.0
1.0	6.7089767e-005	6.9078097e-005	1.9699678e-005	4.7306911e-006
2.0	6.9078097e-005	2.1450144e-004	1.8999699e-004	9.1226313e-005
3.0	1.9699678e-005	1.8999699e-004	4.3252802e-004	4.1681908e-004
4.0	4.7306911e-006	9.1226313e-005	4.1681908e-004	8.9231928e-004

Average error of the Centroidal Scheme = 1.3000668e-004

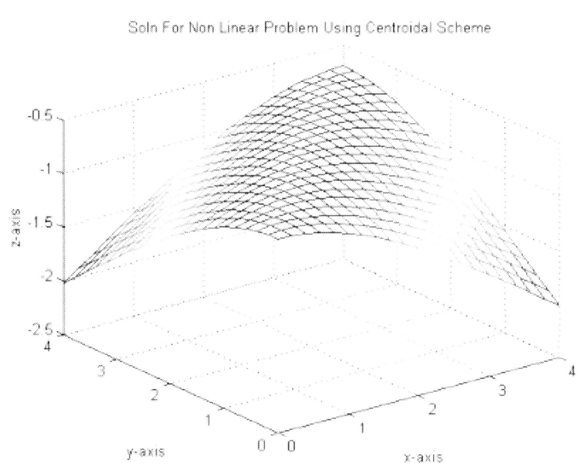

Figure 1: Solution in Graphic form for Problem (9)

Relative Errors for Arithmetic Scheme

Y	X = 1.0	2.0	3.0	4.0
1.0	7.2890713e-005	9.7076691e-005	6.4976523e-005	4.0385671e-005
2.0	9.7076691e-005	2.9467327e-004	3.4128209e-004	2.5890833e-004
3.0	6.4976523e-005	3.4128209e-004	8.0424181e-004	9.5132232e-004
4.0	4.0385671e-005	2.5890833e-004	9.5132232e-004	2.1149666e-003

Average error of the Arithmetic Scheme = 2.5268253e-004

For grid sizes h = 0.025, 0.100, 0.200 and 0.500 we obtained the following results

	h = 0.025	h = 0.100	h = 0.200	h = 0.500
Average error of Centroidal scheme	3.2117909e-005	5.3298255e-004	2.2472913e-003	1.6844006e-002
Average error of Arithmetic scheme	6.2298718e-005	1.0387387e-003	4.3798764e-003	3.1510452e-002

We consider the derivative linear Goursat problem below:

$$u_{xy} = \frac{u_x + u_y + u}{3}$$

$$\begin{aligned} &u(x,0) = e^x \\ &u(0,y) = e^y \\ &0 \le x \le 4,\ 0 \le y \le 4 \end{aligned} \tag{10}$$

The analytical solution for (10) can be found in Day (1966).

A computer program to implement the Centroidal Scheme was developed. For the derivative linear Goursat problem (10) with h = 0.0125, we obtained

Relative Errors for Centroidal Scheme

Y	X = 0.5	1.0	1.5	2.0
0.5	7.6361544e-004	1.3389312e-003	1.7720348e-003	2.0978319e-003
1.0	1.3395230e-003	2.3944425e-003	3.2231452e-003	3.8726349e-003
1.5	1.7735183e-003	3.2244443e-003	4.4061765e-003	5.3647931e-003
2.0	2.1003223e-003	3.8756082e-003	.3667861e-003	6.6123506e-003

Average error of the Centroidal Scheme = 2.1333016e-003

Relative Errors for Arithmetic Scheme

X

Y	0.5	1.0	1.5	2.0
0.5	7.6521334e-004	1.3420271e-003	1.7764802e-003	2.1034593e-003
1.0	1.3420271e-003	2.3994449e-003	3.2305250e-003	3.8822038e-003
1.5	1.7764802e-003	3.2305250e-003	4.4153686e-003	5.3769759e-003
2.0	2.1034593e-003	3.8822038e-003	5.3769759e-003	6.6261263e-003

Average error of the Arithmetic Scheme = 2.1377548e-003

For grid sizes h = 0.005, 0.01, 0.02 and 0.025 we obtained the following results

	h = 0.005	h = 0.01	h = 0.02	h = 0.025
Average error of Centroidal scheme	8.5423982e-004	1.7072556e-003	3.4095780e-003	4.2588655e-003
Average error of Arithmetic scheme	8.5495225e-004	1.7101055e-003	3.4209800e-003	4.2766835e-003

Conclusions

In this paper we have developed a new scheme for the Goursat problem. We have implemented the scheme on a non linear Goursat problem and a derivative linear Goursat problem and found the scheme to be more accurate than the Arithmetic Scheme (for these two problem). However, we note that this new scheme is non linear and thus, even for, linear Goursat partial differential equation, would require iteration.

Acknowledgement

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